Fractal dimensions of lava-flow margins

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Abstract. Fractals are objects, either real or mathematical, that look the same at all scales. They exhibit self-similar behavior or characteristic patterns found to repeat at progressively smaller scales. Lava-flow margins are fractals. Using a technique called structured-walk, the fractal dimension of a flow margin can be determined. A fractal dimension, D, is the measure of a curve’s deviation from linearity with a straight line defined as D = 1. Hawaiian a’a lava-flow margins have fractal dimensions of D = 1.05-1.09 whereas the dimension of pahoehoe lava-flow margins is D = 1.14-1.23. Fractal dimensions of lava flow-lobe margins can be determined by the structured-walk technique and/or box-counting dimension method. Comparing the fractal dimensions determined by these methods from lava flows on Mars to established, well documented Hawaiian lava flows can constrain the emplacement style of Martian lava flows.

1. Introduction

With the advent of new technology used to image the surface of Mars, more questions have been raised concerning the nature of volcanism on the fourth planet from the Sun. The composition of Martian lava flows is thought to be consistent with that of basalt [e.g. Greeley and Spudis, 1981; McSween, 1985; Mouginis-Mark et al., 1993]; however, emplacement styles of the flows remain unknown [e.g. Zimbelman, 1985; Mouginis-Mark et al., 1993; Bruno et al., 1992, 1994]. One way to constrain the emplacement styles of effusive lava flows is through the use of fractal dimensions.

2. Background

Fractals are objects that look the same at all scales [Mandelbrot, 1967, 1983]. As seen in Figure 1, the lava-flow margins look nearly the same at 20 m and 1 km. This is known as self-similar behavior and is characteristic of fractals. Because lava-flow margins are fractals [Bruno et al., 1992], they therefore have a fractal dimension.

A fractal dimension, D, is a measure of tortuosity (repeated twists, bends, or turns) that is dimensionless [Turcotte, 1991]. Mandelbrot [1967, 1983] followed by García [1991] established that a straight line has a fractal dimension of D = 1. The more curves in a line, the larger the fractal dimension, but it will never exceed a fractal dimension of 2. A line has only two dimensions (length and width); thus its fractal dimension must be 1≤D≤2.

Figure 1. Margin of a typical pahoehoe flow from the 1972 eruption of Mauna Ulu, Kilauea volcano, Hawaii, with a small section enlarged to show self-similarity. The similar shapes of the entire flow margin and the enlarged section at different scales suggest fractal behavior [Bruno et al., 1994].

Bruno et al. [1992, 1994] established the fractal dimensions of Hawaiian a’a and pahoehoe flows erupted in 1972 from Mauna Ulu, Kilauea volcano, Hawaii, using the structured-walk technique. The fractal dimension of a’a flow margins is D = 1.04-1.09; for pahoehoe, D = 1.14-1.23. Bruno et al. [1992] did the only calculations of fractal dimensions on Mars to date. The fractal dimension of two lava-flow margins in Elysium Mons were calculated; one has a fractal dimension of D = 1.06, and is interpreted to be a’a-like in morphology. The other flow margin has a fractal dimension of D = 1.19, and is interpreted to have a pahoehoe-like morphology. However, Elysium Mons is not representative of all the types of volcanoes found on Mars [Mouginis-Mark et al., 1993]. Therefore, the fractal dimensions calculated for flow-lobe margins at Elysium Mons cannot necessarily be applied to lava-flow margins at other Martian volcanoes.

3. Methodology

3.1 The Structured-Walk technique

After flow lobes have been mapped within a flow field, the fractal dimension of the flow margin can be determined. Two different methods can be used to...
calculate the fractal dimension of a lava-flow margin. One is the structured-walk technique coined by Richardson [1961]. This method involves measuring the margin of a flow by “walking” rods of different lengths along the flow margin (Figure 2a). The length of the rod used to estimate the length of the margin determines the number of rods needed to walk the margin of the flow.

For example, take a circle which will represent a hypothetical lava flow. Using 4 rods of equal length to outline the boundary of the flow margin creates a square that lies on the boundary of the circle (Figure 4a). These few rods at a longer length do not approximate the boundary of the circle very well. The next step would be to use more rods of shorter length. Using 8 rods of equal length defines the boundary of the circle better than 4 rods do (Figure 4b), but still does not completely outline the boundary. Using more rods at shorter and shorter lengths (Figure 4c, d) approximates the boundary of the circle better than fewer rods at longer lengths. Thus an inverse relationship exists between the length of the rod and how many rods are needed to calculate the fractal dimension. The fractal dimension is determined by plotting the log of the flow margin length vs. the log of the rod length (Figure 2b). This is known as a Richardson plot [1961]. The absolute value of the slope of the line is the fractal dimension.

3.2 The Box-Counting dimension method

The other method that can be used to determine the fractal dimension is the box-counting dimension method. Placing boxes of equal area side by side along the estimated length of the flow margin allows for the calculation of the dimension (Figure 3a). The number of boxes needed is determined by the size of each box. An inverse relationship exists between the size of the box and how many boxes are needed to outline a flow margin. Using a Richardson plot (Figure 3b), or plotting the log of the number of boxes needed to cover the length of the flow margin vs. the log of the box length, and taking the absolute value of the slope determines the fractal dimension.

Figure 2. An example of the structured-walk technique. a) the structure of a freely drawn random rugged profile and construction procedure used for estimating the perimeter of the profile within the equivalent polygon. b) the Richardson plot for various perimeter estimates of the profile [modified from Fig. 2.4 in Kaye, 1994].

Figure 3. An example of the box-counting dimension method. a) construction procedure for estimating the number of boxes needed to cover a coastline. b) the Richardson plot for various size box estimates of the profile [modified from Fig. 3.5 and 3.6 in Addison, 1997].
Figure 4. An example of the structured-walk technique; the black outline of the circle represents a lava-flow margin. a) the structure-walk technique using 4 rods of equal length. b) the structure-walk technique using 8 rods of equal length. c) the structure-walk technique using 16 rods of equal length. d) the structure-walk technique using 32 rods of equal length.

4. Application

The box-counting dimension method can be used in conjunction with the structured-walk technique to more accurately determine the fractal dimension of the lava-flow margins. The structured-walk technique can be implemented on Mars using data from Viking Orbiter (100-230 m/pixel) and Mars Orbiter Laser Altimeter (MOLA) (30-50 m/pixel) images as the longer rod lengths (>1 km) and Mars Observer Camera (MOC) (1.5-12 m/pixel) for shorter rod lengths (≤1 km). One way to apply the box-counting dimension method to Martian flows is through the use of an ArcView script written by Louise M. Norman [1999, 2000].

Calculating the fractal dimensions of lava-flow margins with known eruption parameters such as effusion rate, eruption duration, and emplacement styles is extremely important. Comparing the fractal dimensions calculated for the margins of Hawaiian flows with known eruption parameters, to those fractal dimensions calculated on Mars can aid in determining the emplacement style of Martian flows. Morphologies can be better constrained with the knowledge of emplacement style. If the fractal dimensions changes, this could indicate a change in time or place of the emplacement style of a flow such as a transition from a pahoehoe-like to an a‘a-like morphology.

References


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