Earthquake Magnitudes and Moments

Using information about wave amplitudes to learn about the earthquake size.

Need to correct for decrease with distance

$M = \log(A/T) + F(h,\Delta) + C$

A is the amplitude of the signal
T is the dominate period
F is the distance/depth correction
C is a regional scale factor

Local magnitudes M_L - introduced by Richter for southern California earthquakes recorded on a specific *Wood-Anderson* seismograph.

$M_L = \log A + 2.76\log\Delta - 2.48$

Body wave magnitude m_b and surface wave magnitude M_s

$m_b = \log(A/T) + Q(h,\Delta)$

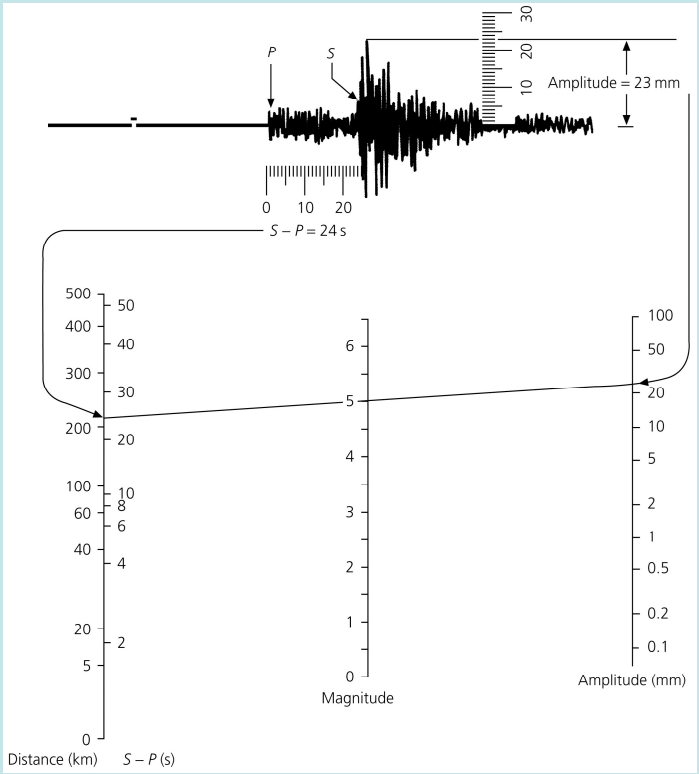
Where A is the ground motion in micron after the affects of the seismometer are removed

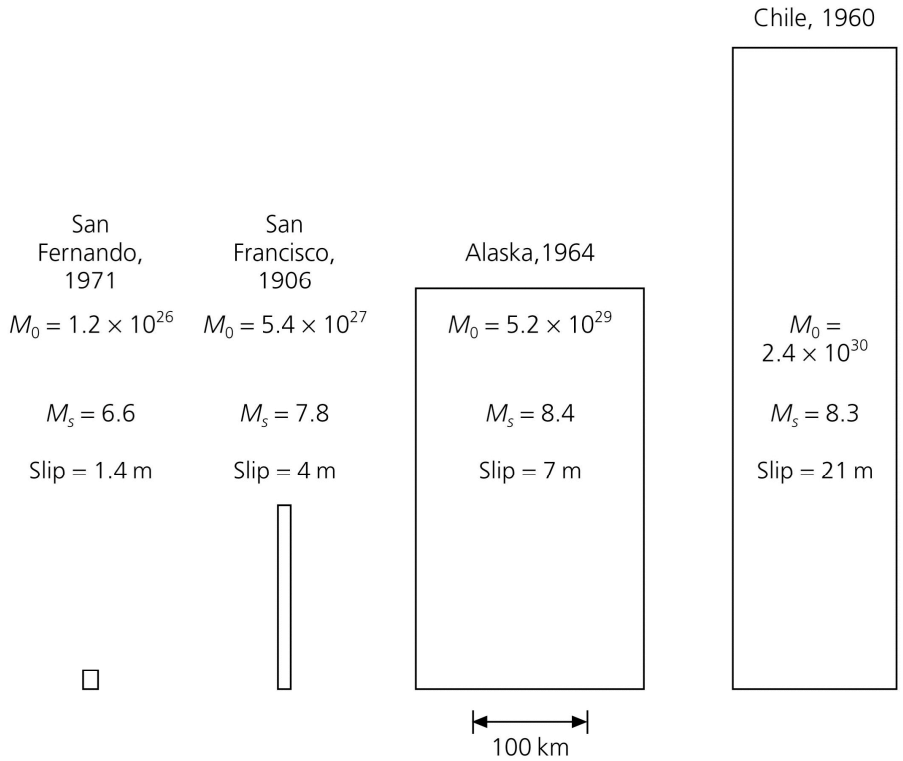
$M_S = \log(A/T) + 1.66\log\Delta + 3.3$

Problems with this approach to estimating earthquake magnitudes:

- no direct connection to physics of the earthquake
- variable range of magnitudes (for the same event as a function of direction, or for different magnitude scales for a given event)
- body and surface wave magnitudes do not correctly reflect the size of large earthquakes

Example of determining M_L from a seismic recording.





earthquake	Body Wave magnitude m_b	Surface Wave magnitude M_S	Fault Area (km^2)	Slip (m)	Moment M_0 (dyne-cm)	Moment magnitude M_w
Truckee, 1966	5.4	5.9	10x10	0.3	8.3×10^{24}	5.9
San Fernando,1971	6.2	6.6	20x14	1.4	1.2×10^{26}	6.7
Loma Prieta,1989	6.2	7.1	40x15	1.7	3.0×10^{26}	6.9
Alaska	6.2	8.4	500x300	7	5.2×10^{29}	9.1

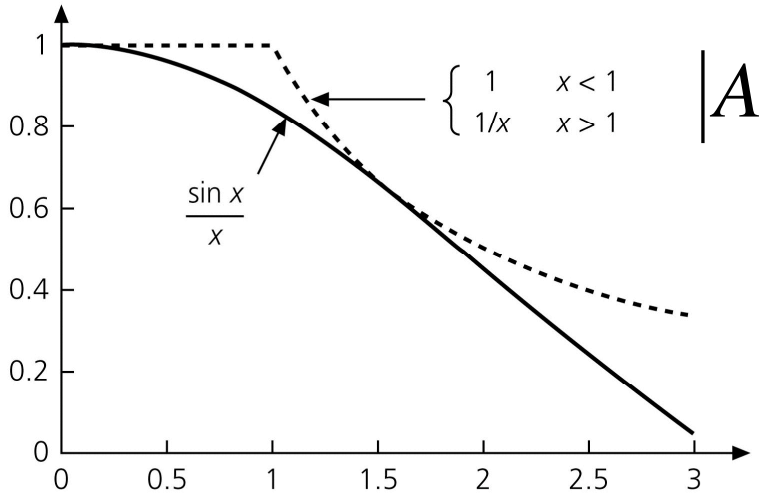
$$M_w = \frac{\log M_0}{1.5} - 10.73$$

Uncertainties:
Uncertainties for historic earthquakes are quite large
-seismic moment estimates vary by ~25% even when modern seismic data is available
-fault length estimates for 1906 San Francisco earthquake vary from 300 to 500 km
-fault dimensions is essentially inferred from the depths of more recent earthquakes and geodetic data
-different techniques can yield different estimates (body wave magnitude, vs. surface wave magnitudes, vs geologic estimates).

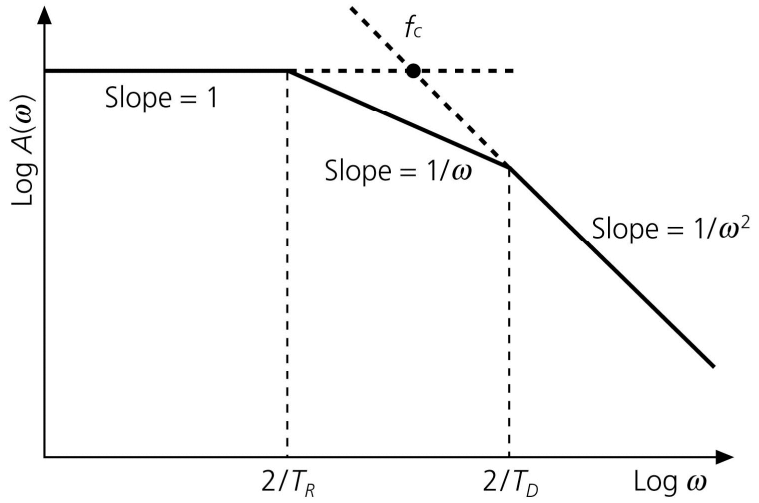
Frequency variations can explain differences between magnitudes and their saturation.

Scaling Laws

Source spectra varies with earthquake size, the source signal is the product of the seismic moment and two sinc terms where T_R and T_D are the rupture and rise times:



$$\left|A(\omega)\right| = M_0 \left|\frac{\sin(\omega T_R/2)}{\omega T_R/2}\right| \left|\frac{\sin(\omega T_D/2)}{\omega T_D/2}\right|$$



Two corner frequencies $2/T_R$ and $2/T_D$. The the spectrum flat at frequencies less than the first.

The spectrum is parameterized by 3 factors, seismic moment, rise time, and rupture time. If effects of fault with are significant, it would add a third corner.

Scaling Laws

The approximated source spectra is divided into three regions corresponding to different frequency ranges.

$$|A(\omega)| = \begin{cases} \log M_0 & \omega < 2/T_R \\ \log M_0 - \log(T_R/2) - \log\omega & 2/T_R < \omega < 2/T_D \\ \log M_0 - \log(T_R T_D/4) - 2\log\omega & 2/T_D < \omega \end{cases}$$

At low frequencies, the seismic moment is the scale factor for the spectral amplitude:

$$M_0 = \mu \overline{D} S = \mu \overline{D} f L^2$$

The rupture time for the rupture to propagate along the fault can be approximated in terms of the shear wave speed:

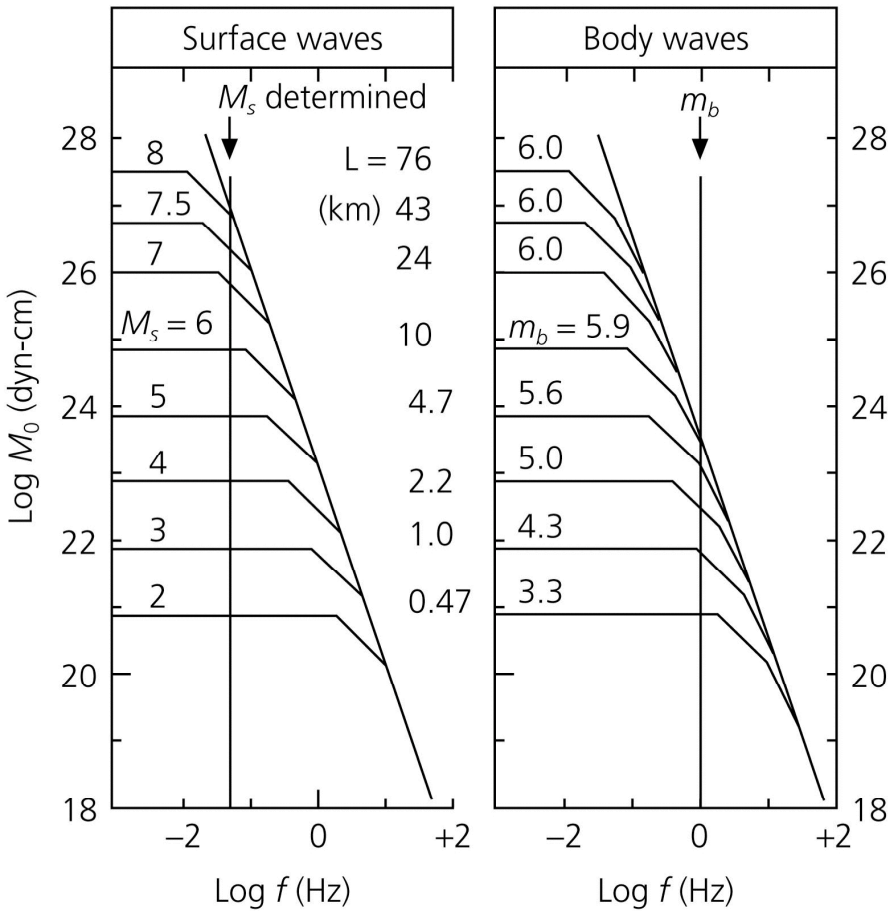
$$T_R = L/v_R = L/.7\beta$$

The rise time for the dislocation to reach its full value at any point along the fault has been approximated as:

$$T_D = \mu \overline{D}/(\beta \Delta\sigma)$$

Size Saturation

As the fault length increases, seismic moment, rupture time, and rise time increase. These factors all push the corner frequencies to lower values.



The moment M_0 is the zero frequency which rises as the earthquake grows larger.

The surface wave magnitude M_S is measured at a period of 20 s and accordingly depend on the spectral amplitude at this period.

However, for large moments, 20 s is to the right of the first corner frequency so M_S does not increase at the same rate as the moment. M_S saturates around 8.2.

Using seismic waves to estimate stress change:

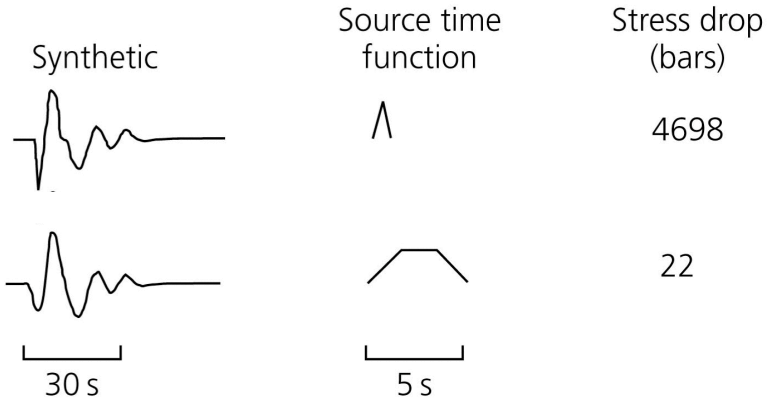
Earthquake slip (***D***), occurs on a fault with characteristic dimension (***L***) resulting in a strain change of around:

$$e_{xx} = \frac{\partial u_x}{\partial x} = \frac{\overline{D}}{L}$$ and a average stress drop of: $\Delta\sigma = \mu\overline{D} / L$

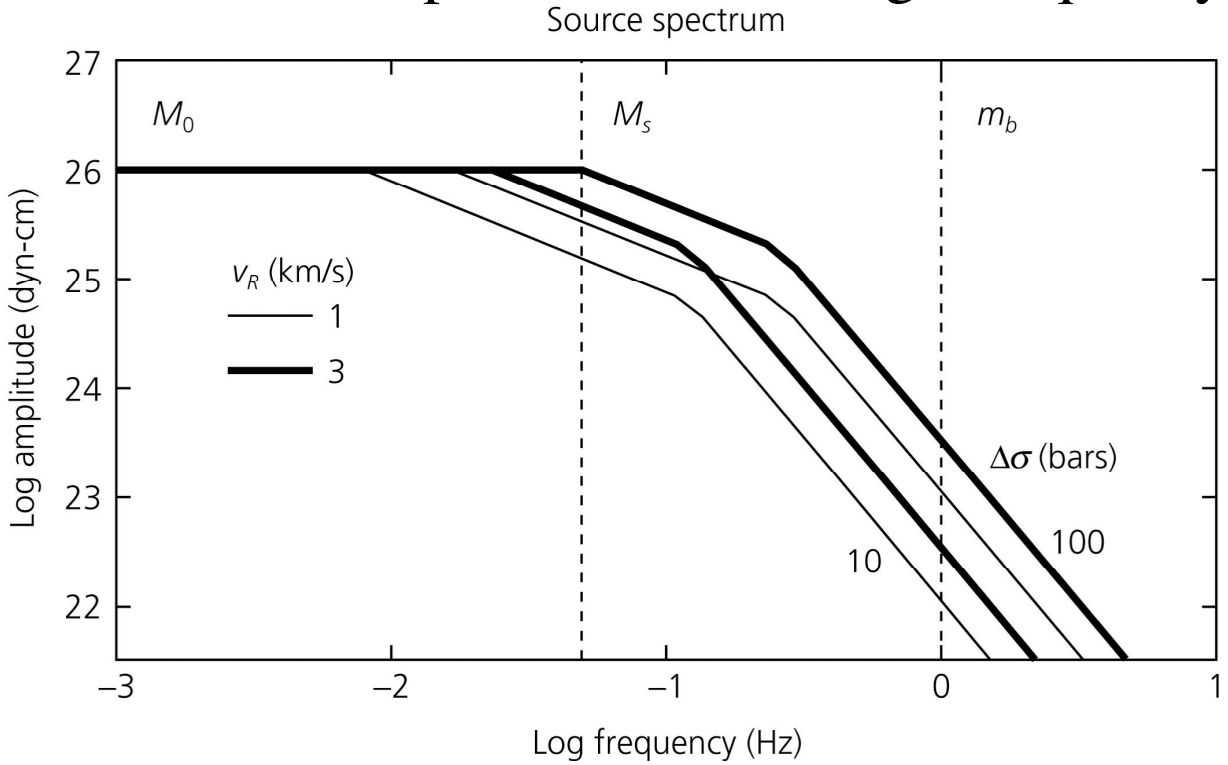
Stress drop during an earthquake is independent of M_0

Slip is proportional to fault length-- seismic waves can be used to model stress drop.

Modeling stress drop depends on the fault dimension and source time functions- uncertainty in the fault dimensions can lead to uncertainties in stress drop.



Energy radiated out from an earthquake varies with frequency. For two equivalent earthquakes with the same rupture velocity, the one with the lower stress drop will have less high-frequency radiation.



From a source spectrum view, earthquake magnitudes saturate because the stress drop is essentially constant as earthquake size increases.

Larger-moment earthquakes have longer faults and hence lower corner frequencies.

Stress drop averaged over the fault is approximately:

$$\Delta\sigma = \mu\overline{D} / L$$

With average slip from the seismic moment:

$$\overline{D} = cM_0 /(\mu L^2)$$

c is a factor related to the faults shape

For a circular fault of radius R

$$\Delta\sigma = \frac{7}{16}\frac{M_0}{R^3}$$

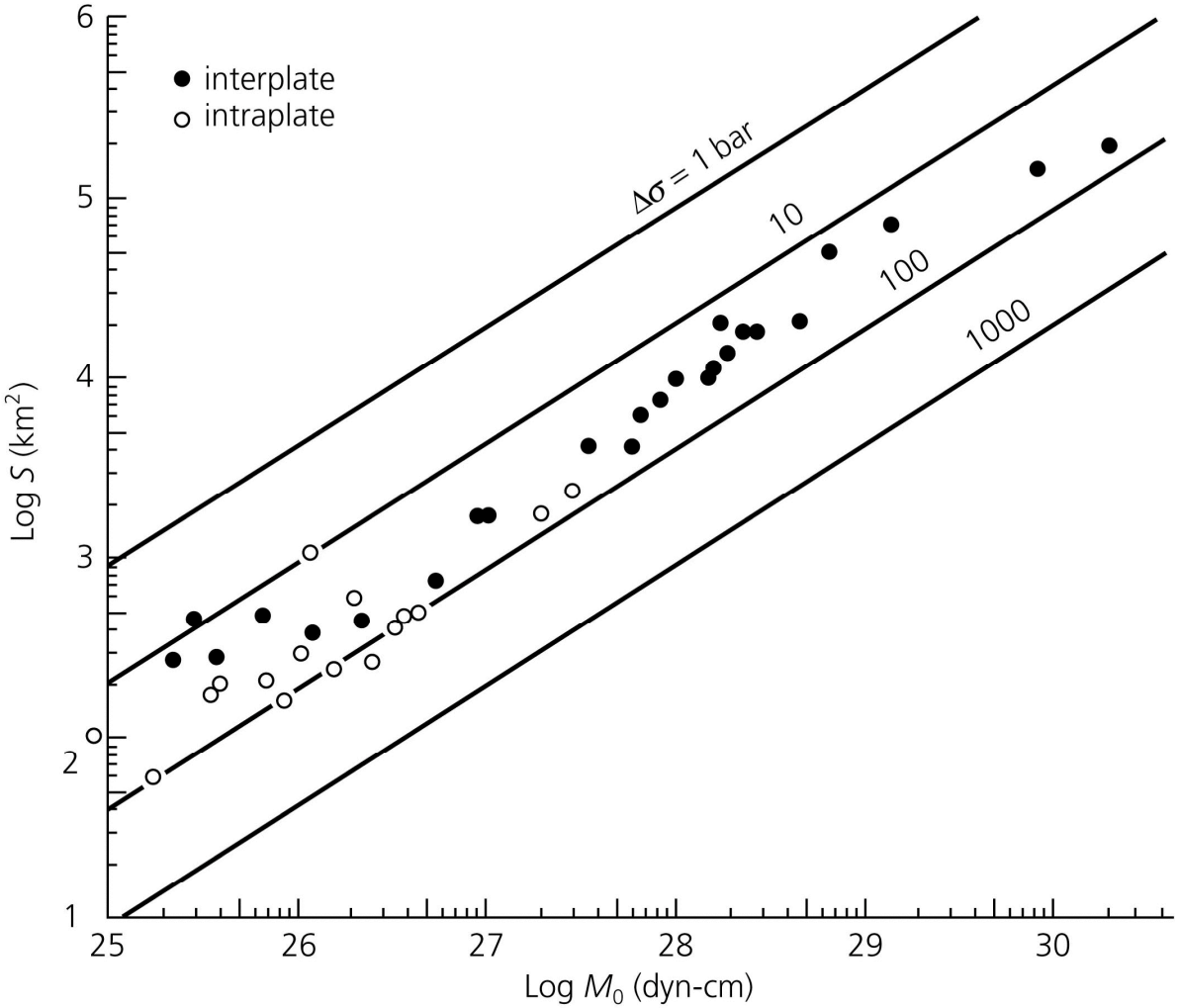
While a rectangular fault (length L, width *w*) with strike-slip motion will have a stress drop of :

$$\Delta\sigma = \frac{2}{\pi}\frac{M_0}{w^2 L}$$

For dip-slip motion on a rectangular fault (if $\lambda=\mu$):

$$\Delta\sigma = \frac{4(\lambda+\mu)}{\pi(\lambda+2\mu)}\frac{M_0}{w^2 L} = \frac{8}{3\pi}\frac{M_0}{w^2 L}$$

Earthquake stress drop studies typically find values in the range of 10 to 100 bar. Stress drop typically remains constant over 5 orders of moment magnitude, but differences exists between interplate and intraplate events.



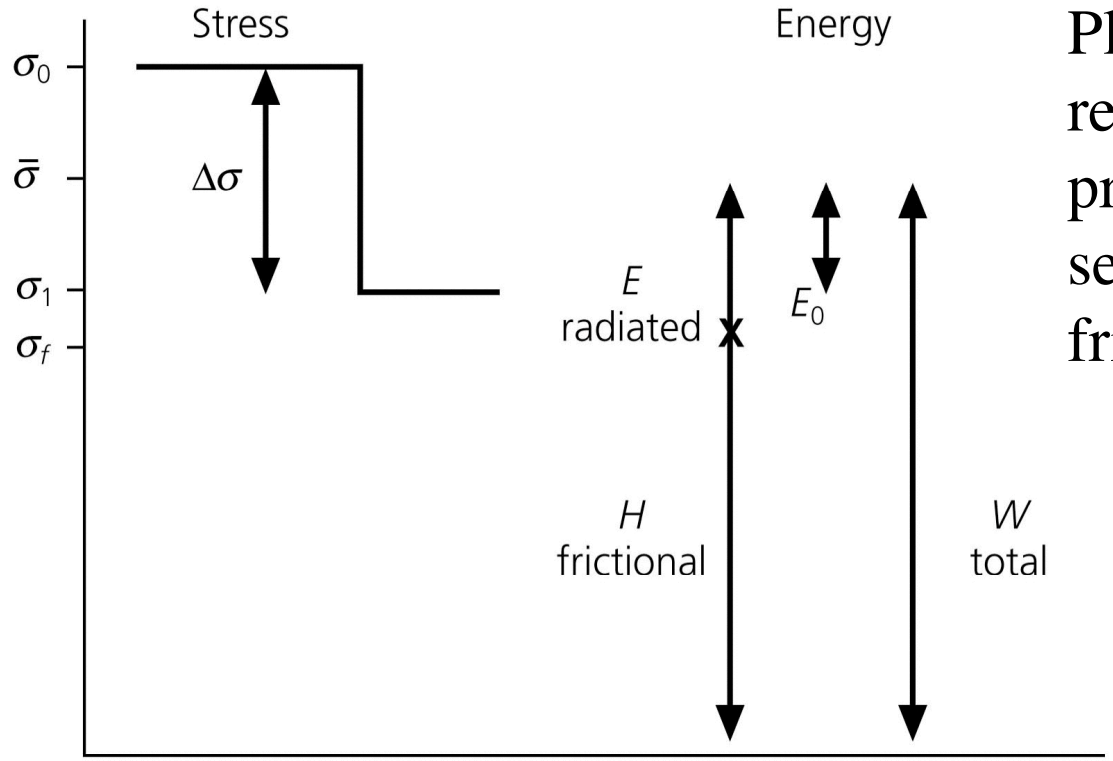
Radiated Seismic energy E - differences between the stress before and after faulting

$$\Delta\sigma = \sigma_0 - \sigma_1$$

average stress $\dot{\sigma} = \sigma_1 + \Delta\sigma/2$ σ_f= frictional stress

The lower bound of radiated seismic energy is:

$$E_0 = (\Delta\sigma/2)\overline{D}S = (\Delta\sigma/2\mu)M_0$$



Plot of total energy released (W) and its proportions radiated seismically (E) and frictionally (H)

Assuming:

$$\mu = 5 \times 10^{11} \text{ dyne/cm}^2$$

$$\Delta\sigma = 50 \text{ bars}$$

$$E_0 = M_0 / 2 \times 10^4 \text{ in ergs}$$

The definition of moment magnitude gives:

$$\log E_0 = \log M_0 - 4.3$$

$$\log M_0 = 1.5 M_w - 16.1$$

$$\log E_0 = 1.5 M_w - 11.8$$

Increase in magnitude of 1 unit from 5 to 6
-Corresponds to an increase in radiated energy by $10^{1.5}$ or ~32 times

Magnitude 7 releases 1000 times more energy than a magnitude 5